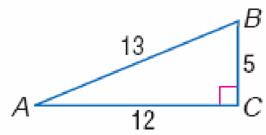
Trigonometry

Concept Summary

- Trigonometric ratios can be used to find measures in right triangles.

Find sin *A*, cos *A*, and tan *A*. Express each ratio as a fraction and as a decimal.

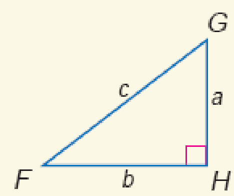


$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$ $= \frac{BC}{AB}$ $= \frac{5}{13}$ or about 0.38

$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$ $= \frac{AC}{AB}$ $= \frac{12}{13}$ or about 0.92

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$ $= \frac{BC}{AC}$ $= \frac{5}{12}$ or about 0.42

Exercises **Use $\triangle FGH$ to find sin *F*, cos *F*, tan *F*, sin *G*, cos *G*, and tan *G*. Express each ratio as a fraction and as a decimal to the nearest hundredth.** *See Example 1 on page 365.*



21. $a = 9, b = 12, c = 15$ **22.** $a = 7, b = 24, c = 25$

Find the measure of each angle to the nearest tenth of a degree.
See Example 4 on pages 366 and 367.

23. $\sin P = 0.4522$ **24.** $\cos Q = 0.1673$ **25.** $\tan R = 0.9324$

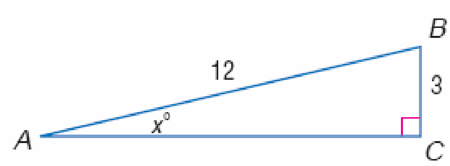
Angles of Elevation and Depression

Concept Summary

- Trigonometry can be used to solve problems related to angles of elevation and depression.

A store has a ramp near its front entrance. The ramp measures 12 feet, and has a height of 3 feet. What is the angle of elevation?

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x° = \frac{BC}{AB}$ — $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin x° = \frac{3}{12}$ — $BC = 3$ and $AB = 12$

$x = \sin^{-1}\left(\frac{3}{12}\right)$ — Find the inverse.

$x \approx 14.5$ — Use a calculator.

The angle of elevation for the ramp is about 14.5°.

Exercises **Determine the angles of elevation or depression in each situation.**
See Examples 1 and 2 on pages 371 and 372.

26. An airplane must clear a 60-foot pole at the end of a runway 500 yards long.

27. An escalator descends 100 feet for each horizontal distance of 240 feet.

28. A hot-air balloon descends 50 feet for every 1000 feet traveled.

29. **DAYLIGHT** At a certain time of the day, the angle of elevation of the sun is 44°. Find the length of a shadow cast by a building that is 30 yards high.

30. **RAILROADS** A railroad track rises 30 feet for every 400 feet of track. What is the measure of the angle of elevation of the track?

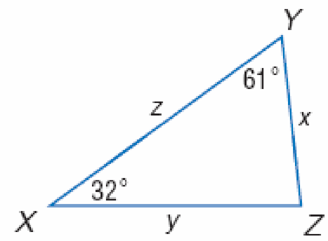
The Law of Sines

Concept Summary

- To find the measures of a triangle by using the Law of Sines, you must either know the measures of two angles and any side (AAS or ASA), or two sides and an angle opposite one of these sides (SSA) of the triangle.
- To solve a triangle means to find the measures of all sides and angles.

Solve $\triangle XYZ$ if $m\angle X = 32$, $m\angle Y = 61$, and $y = 15$. Round angle measures to the nearest degree and side measures to the nearest tenth.

Find the measure of $\angle Z$.



$m\angle X + m\angle Y + m\angle Z = 180$ Angle Sum Theorem

$32 + 61 + m\angle Z = 180$ $m\angle X = 32$ and $m\angle Y = 61$

$93 + m\angle Z = 180$ Add.

$m\angle Z = 87$ Subtract 93 from each side.

Since we know $m\angle Y$ and y, use proportions involving sin Y and y.

To find x:		To find z:
$\frac{\sin Y}{y} = \frac{\sin X}{x}$	Law of Sines	$\frac{\sin Y}{y} = \frac{\sin Z}{z}$
$\frac{\sin 61°}{15} = \frac{\sin 32°}{x}$	Substitute.	$\frac{\sin 61°}{15} = \frac{\sin 87°}{z}$
$x \sin 61° = 15 \sin 32°$	Cross products	$z \sin 61° = 15 \sin 87°$
$x = \frac{15 \sin 32°}{\sin 61°}$	Divide.	$z = \frac{15 \sin 87°}{\sin 61°}$
$x \approx 9.1$	Use a calculator.	$z \approx 17.1$

Exercises **Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.**
See Example 1 on page 378.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$.

32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$.

Solve each $\triangle ABC$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth. *See Example 2 on pages 378 and 379.*

33. $a = 15$, $b = 11$, $m\angle A = 64$

34. $c = 12$, $m\angle C = 67$, $m\angle A = 55$

35. $m\angle A = 29$, $a = 4.8$, $b = 8.7$

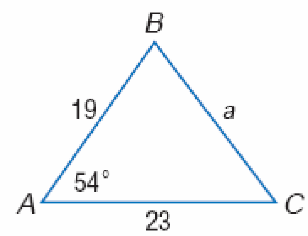
36. $m\angle A = 29$, $m\angle B = 64$, $b = 18.5$

The Law of Cosines

Concept Summary

- The Law of Cosines can be used to solve triangles when you know the measures of two sides and the included angle (SAS) or the measures of the three sides (SSS).

Find a if $b = 23$, $c = 19$, and $m\angle A = 54$.



Since the measures of two sides and the included angle are known, use the Law of Cosines.

$a^2 = b + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$	$b = 23$, $c = 19$, and $m\angle A = 54$
$a = \sqrt{23^2 + 19^2 - 2(23)(19) \cos 54°}$	Take the square root of each side.
$a \approx 19.4$	Use a calculator.

Exercises **In $\triangle XYZ$, given the following measures, find the measure of the missing side.** *See Example 1 on page 385.*

37. $x = 7.6$, $y = 5.4$, $m\angle Z = 51$

38. $x = 21$, $m\angle Y = 73$, $z = 16$

Solve each triangle using the given information. Round angle measure to the nearest degree and side measure to the nearest tenth.
See Example 3 on pages 386 and 387.

39. $c = 18$, $b = 13$, $m\angle A = 64$

40. $b = 5.2$, $m\angle C = 53$, $c = 6.7$