Practical Trigonometry Math by MiLearn

Introduction

Here are some ways that everyday work brings math skills into play. Perhaps these examples will help shed some light on some of the more confusing elements of math, in particular, trigonometry.

Do you drag your way through your math courses? Do you wonder when and how you will ever use trigonometry?

Relating mathematics to real life makes it more interesting to you, but more importantly, it helps you understand how invaluable it is.

Whether you want to build a skateboard ramp, a stairway, or a bridge, you can't escape trigonometry.

Objectives

We will show you how to use trigonometry in work situations that require an understanding of specific math concepts to get the job done.

In this short online tutorial you will learn:

- how to use sine, cosine and tangent functions in everyday life
- Solve real-life problems
- Explain the importance of understanding trigonometry within the right triangle
- Find relations between the opposite side, the adjacent side and the hypotenuse of a right triangle
- Work with the sin and $\sin^{-1}$ keys on a calculator to find the measure of an angle.

A bit about Trigonometry

Origins and Uses

Trigonometry is an extension of geometry. It helps you understand and work with angles of elevation or cyclical patterns like waves of sound or light. You often use trigonometry with geometry to calculate the relations between angles and circles.

Originally used by surveyors, ship captains, and astronomers to calculate distances, trigonometry has widespread uses today. Engineering, carpentry, surveying, physics, navigation are just a few fields where it is used on a regular basis. Professionals in the natural sciences, such as geography, also use trigonometry; it helps to predict weather patterns and measure earth tremors.

You can also use trigonometry in your daily activities, to find the best angle for a stairway or the ideal height and width of steps, for example. Some hikers also use it to estimate their distance from the mountaintop.

The word "trigonometry" derives from the Greek "trigonon" and "metron" ("tri" for "three," "gonia" for "angle" and "metron" meaning "to measure").

So trigonometry literally means the measurement of three angles (or triangles).

The Pythagorean Theorem

In trigonometry, you often base your calculations on the Pythagorean Theorem. This formula helps you find the length of one side of a right triangle when you already know the measurement of the other two sides.

Expressed in plain language, the Pythagorean Theorem states that if you measure the longest side of a right triangle, the hypotenuse, and multiply that number by itself, ***the product will equal the sum of the squares of the other two sides*****.**

This formula is proven mathematically. In other words, the logic behind it is perfectly sound. If you test it in real-life situations, you will find that it works.

In the working world, the Pythagorean Theorem is applied quite frequently. Engineers use it to perform calculations on even the most complex structures.

Remember!

The word "hypotenuse" derives from the Greek "hypoteinousa" meaning "to stretch".))

In mathematical terms, the Pythagorean Theorem states that the square of the hypotenuse equals the square of the other two sides of a triangle.

$$c^2 = a^2 + b^2.$$

Pythagoras, a Greek philosopher and mathematician, did not father the theorem himself. The Babylonians used it for over a thousand years before him, and the Chinese, Indians, and Mayas knew of it.

However, Pythagoras was probably first to prove the theorem, and he taught it at the school he founded in Italy.

For a biography of Pythagoras, visit this website: http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html

A quick animation of the Pythagoras Theorem

http://www.usna.edu/MathDept/mdm/pyth.html.

Illustrating the Pythagorean Theorem

Let’s start putting theory into practice. Imagine yourself as a structural engineer for a consulting company. Your firm is designing an innovative bridge to cross the St-Lawrence River in hope of easing the ever-so-present traffic problem in Montreal.

The architect has specified that the bridge must be sleek, slender, and aesthetic. In addition, the navigational clear span of the bridge must be maximized for the passage of ships as Montreal is a port of call.

The head engineer has chosen to design a “cable-stayed bridge”, which he believes would be the best solution.

Remember!

"Cable-stayed bridges look similar to suspensions bridges, but they support the load of the roadway differently.

In cable-stayed bridges, the cables are attached to the towers, which alone bear the load.

In suspension bridges, the cables ride freely across the towers, transferring the load to the anchorages at either end."

For more information, check out the link below…

http://www.pbs.org/wgbh/nova/bridge/meetcable.html.

A typical example of a cable-stayed bridge

As the lead engineer, your specific task is to design the cable supporting the bridge deck. An initial analysis shows that the concrete tower is 150 meters above the road deck and that the support point for the main cable is 100 meters away from the concrete tower.

How long does the cable have to be? To find out, apply the Pythagorean Theorem.

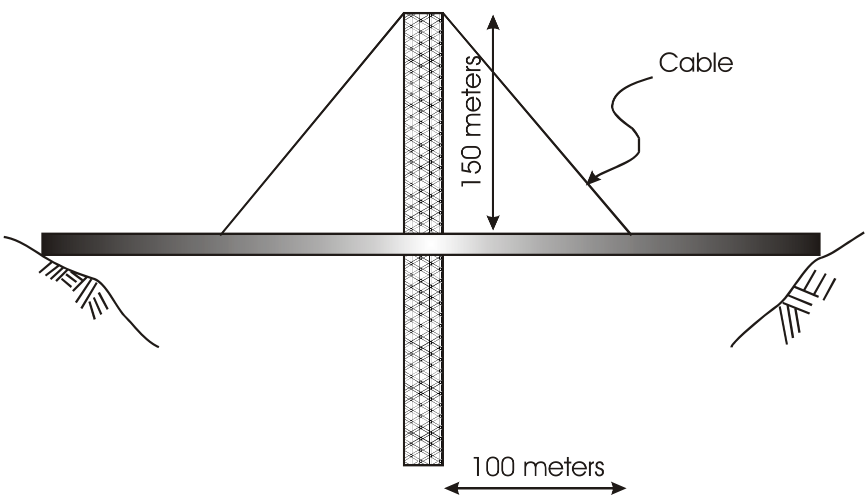
$$c^2 = a^2 + b^2$$
$$a = 100m$$
$$b = 150m$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{100^2 + 150^2}$$
$$c = \sqrt{10000 + 22500}$$
$$c = \sqrt{32500}$$
$$c = 180.28m$$

You would design a cable 180.28 meters long.

Note!

Did you know that a steel wire about 1/10 of an inch thick can support over half a ton without breaking?

Here is a sketch proving the example above



Finding the angle of elevation

As a hydraulic engineer, you are concerned with the amount and velocity of surface runoff resulting from the melting of snow in the springtime. Your consulting firm has asked you to find the grade (angle of inclination) in an area of a park. You know you can do this by using a surveyor's level and casting a horizontal line of sight.

You go to the site and pick two points in the park between which you can measure the length along the slope of the ground. You measure this length and find it to be 100 feet. Using your surveying skills, you then record the elevation at the same two points. It is 7.21 feet and 2.63 feet.

What is the angle of inclination of the ground? The solution to this problem is detailed on the following page.

Note!

> "In mapping a country, surveyors divide it into triangles and mark each corner by a benchmark, which nowadays is often a round brass plate set into the ground, with a dimple in its center, above which the surveyors place their rods and telescopes…"

http://www-istp.gsfc.nasa.gov/stargaze/Strig1.htm

You can find the difference in elevation by subtracting the two elevations.

Difference in elevation = 7.21 – 2.63 = 4.58 feet

You can now use the *sin* relationship to find the angle of inclination, *A*.

$$\text{Sin } A = \frac{\text{measure of opposite side}}{\text{measure of hypotenuse}}$$

Sin A = 4.58 feet / 100 feet

Sin A = 0.0458

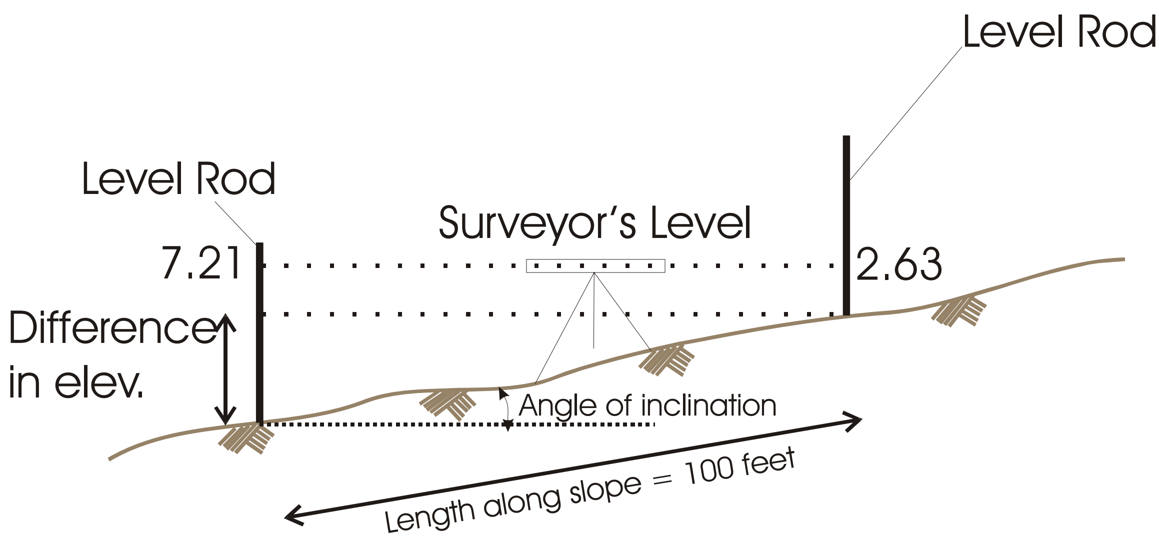
$A = \sin^{-1}(0.0458)$

$A = 2.62°$

You can now report to your boss that the angle of inclination of the ground is 2.62°. This angle will have a direct impact on the amount and velocity of surface runoff due to the snow melting in the spring.

Remember!

$$\text{Sin } A = \frac{\text{measure of opposite side}}{\text{measure of hypotenuse}}$$



Finding the angle of descent

You are an air traffic controller communicating with a plane in flight that will soon land. The pilot tells you he is flying at an altitude of 35,000 feet. You locate the airplane on your radar and tell the pilot that 100,000 feet separate him from the beginning of the runway (that imaginary straight line is the hypotenuse.)

You must tell the pilot which angle of descent he must take to land his plane at the beginning of the runway.

Since you are dealing with the opposite and hypotenuse sides of an angle, use the Sin function.

Sin(A) = 35000ft/100000ft

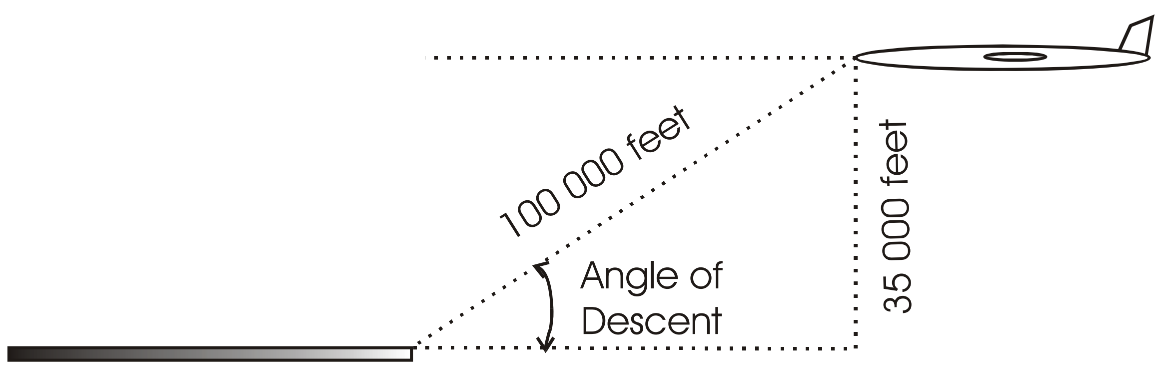
Sin(A) = 0.35

A = $\sin^{-1}(0.35)$

A = 20.5° The pilot must descend at an angle of 20.5°.

Remember!

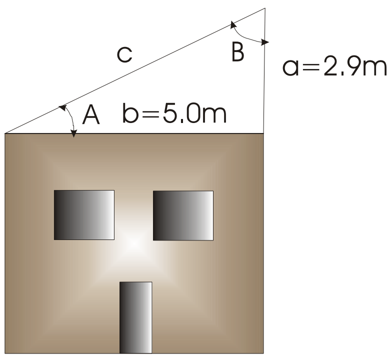
$$\text{Sin } A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$



Designing a Roof

You are considering different possibilities for the framing layout of your roof.

You know a flat roof would be easiest to build, but it would look dull and would not allow for proper drainage during rainstorms or snowmelt. A shed roof is best for drainage and allows the snow to slide off; however, building this type of roof requires much more materials and expertise. You want to find an economical yet practical design. You first consider a roof where *a=2.9m* and *b=5.0 m*.



1. Using the Pythagorean theorem, find the length of the hypotenuse.

a. 8.41 meters

b. 33.41 meters

c. 58 meters

d. 5.8 meters

Answer: d

How??

$$c^2 = a^2 + b^2$$
$$a = 2.9m$$
$$b = 5.0m$$
$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{2.9^2 + 5.0^2}$$
$$c = \sqrt{8.41 + 25}$$
$$c = \sqrt{33.41}$$
$$c = 5.8m$$

2. What is sin A of the triangular roof?

a. b

b. a/c

c. a

d. b/c

e. c

Answer is :b

3. What is sin B of the triangular roof?

a. b

b. a/c

c. a

d. b/c

e. c

Answer is : d

4. What are sin A and sin B?

a. Sin A = 2,9/5,8 = 0.50, Sin B = 5,0/5,8 ≅ 0.86

b. Sin A = 5,0/5,8 ≅ 0.86, Sin B = 2,9/5,8 = 0.50

c. Sin A = 5,8/5,0 ≅ 1.16, Sin B = 5,8/2,9 = 2

d. Sin A = 2,8/8,5 ≅ 0.329, Sin B = 5,9/0,5 = 11.8

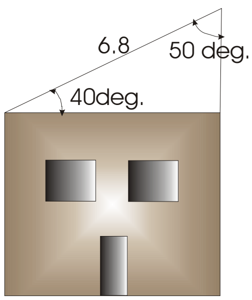
Answer is: a

5. You decide to try a different design.

You know you would like the angle of inclination of the roof to be around 40°.

Knowing that the three angles in a triangle must sum up to 180°, you calculate that the other angle in the triangle must be 50°. You also know that you would like the hypotenuse of the roof to be 6.8 meters.

What is the length of the two sides?



a. One side is 4.37 meters and the other 5.21 meters.

b. One side is 6.8 meters and the other 5.21 meters

c. One side is 4.37 meters and the other 6.8 meters

d. One side is 5.21 meters and the other 6.37 meters.

Answer is: a

a = ___ sin (40^o) = a/6,8

6,8 sin (40^o) = a

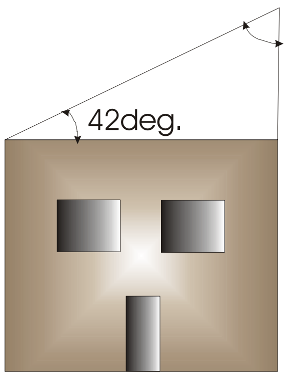
$4{,}37 \cong a$

$b =$ _____ $\sin(50^o) = b/6{,}8$

$6{,}8 \sin(50^o) = b$

$5{,}21 \cong b$

You then decide to try an angle of 42°. You consider this to be optimum for your particular situation.



6. What is the length of side **a**?

a. 48 ^{o}

b. 48 meters

c. 8.10 meters

d. 12.1 meters

Answer is: c

$a =$ _____ $\sin(42^o) = a/12{,}1$

$12{,}1 \sin(42^o) = a$

$8{,}10 = a$

7. What is the measure of the other acute angle (angle A) in the triangle?

a. 30°

b. 40°

c. 48°

d. 58°

Answer is: c

A = _____ $42^{o} + 90^{o} + A = 180$

$A = 180^{o} - 90^{o} - 42^{o}$

$A = 48^{o}$

8. In which if these situations would you use the sine function?

a. You are given the length of the adjacent side and opposite side and are looking to find the angle.

b. You are given the angle and the length of the opposite side and are looking for the length of the hypotenuse.

c. You are given the angle and the length of the hypotenuse and are looking for the length of the adjacent side.

d. You are given the length of the adjacent side and the hypotenuse and are looking for the angle.

Answer is: b

$$\text{Sin } A = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

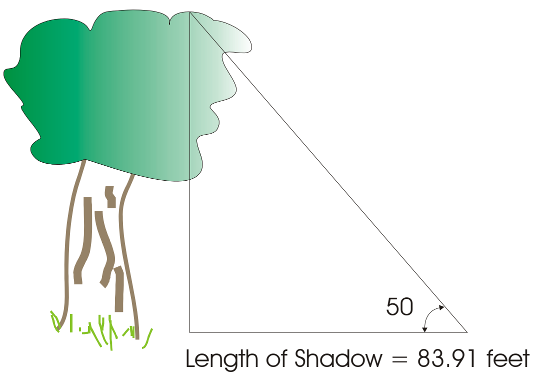
Working with the Tangent Function

Finding the side opposite to the angle (using the tangent function)

You have a dream job managing a spectacular golf club. A tree on the 18th hole is casting a long shadow over the green and the tree may have to be cut down. You contact a company to find out how much they would charge for the job. They charge according to the height of the tree.

You walk over to the tree and realize that instead of directly measuring the tree (it is very tall), you can easily measure the length of its shadow on the ground. You can also estimate the angle at the tip of the shadow.

You do some quick measurements and find the shadow to be 83.91 feet. You also estimate its angle of inclination at 50°. How high is the tree? Use the tangent function for the solution.



$$\tan(50) = \frac{opposite}{adjacent}$$

$$\tan(50) = \frac{height}{83.91\,ft}$$

$$\tan(50) \bullet 83.91 = height$$

$$100\ feet = height$$

So you can tell the contractor the tree is 100 feet tall. (and hope it doesn't cost you too much!)

Remember!

Tan A = $\frac{\text{opposite side of } \angle A}{\text{Adjacent side of } \angle A}$

Finding the side opposite to the angle (using the tangent function)

You are a foreman of a construction site and are expecting a shipment of sand to be delivered. From past experience, you know that the [angle of friction] of sand is 40°.

You have an area about 10m wide on the ground to work with. When the truck arrives, it will dump the sand in this area, and you will stop the dumping when the bottom of the sand pile reaches 10m. Assuming the angle of friction (angle of inclination) of the sand pile is 40°, how high will the sand pile be?

Since you are concerned with the opposite side to the angle as well as the adjacent side to the angle, use the tangent function to solve this problem.

$$\tan(40) = \frac{opposite}{adjacent}$$

$$\tan(40) = \frac{Y}{5m}$$

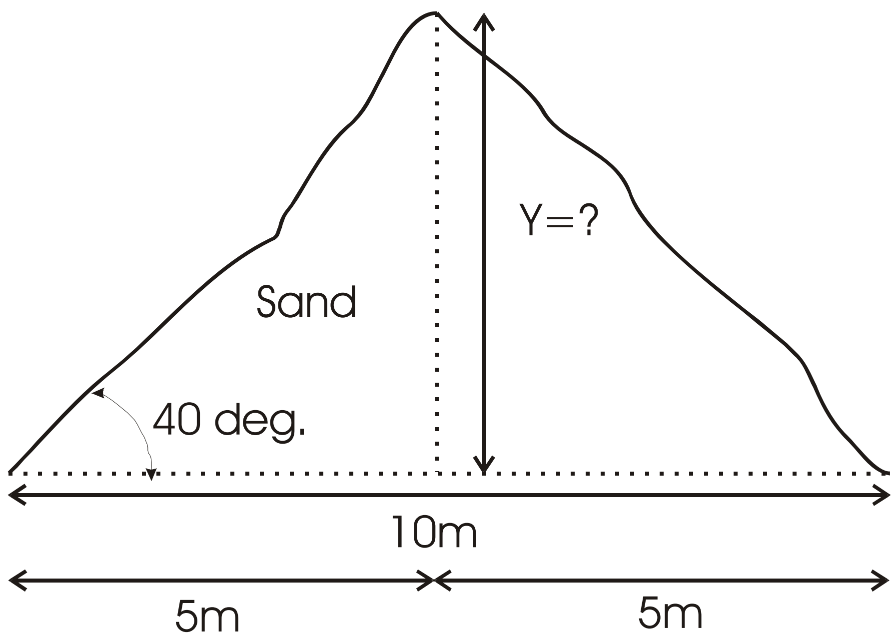
$$\tan(40) \bullet 5 = Y$$

$$4.2m = Y$$

So you know your pile of sand can not be higher than 4.2m if you are limited to an area 10m wide.

Remember!

Tan A = $\frac{\text{opposite side of } \angle A}{\text{Adjacent side of } \angle A}$


Y=?
Sand
40 deg.
10m
5m
5m

Finding the adjacent side to the angle (using the tangent function)

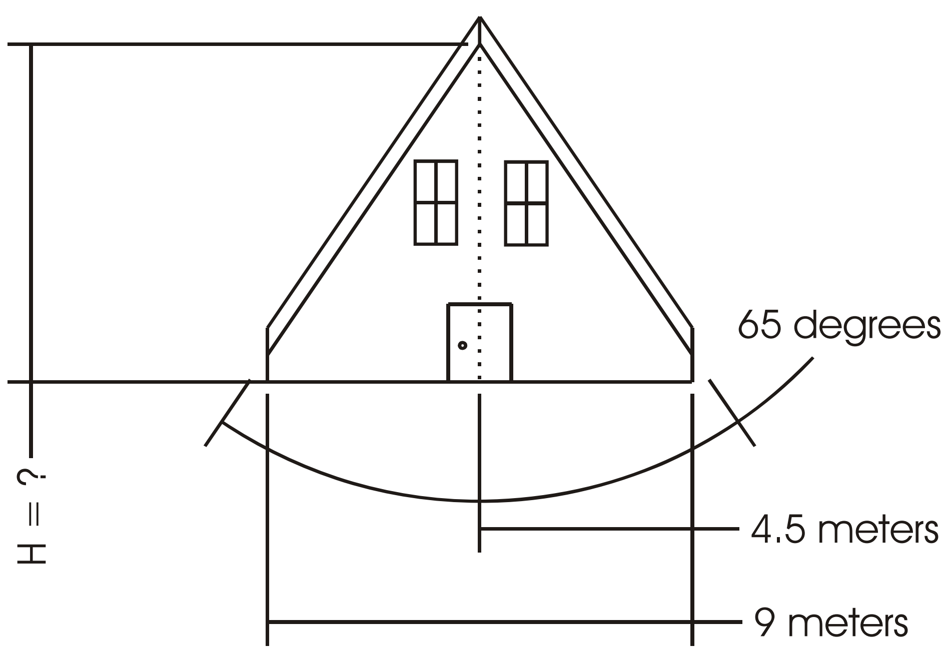
You are a contractor and want to order siding for an A-frame house you are building. You know the house is 9 meters wide and the [included angle] at the peak of the roof is 65°.

How much siding, in square meters, will you need to complete the job on one face of the house (ignoring any reductions in area to allow for windows and doors)? Recall that the area of a triangle is (Base x Height)/2.

This is actually a multi-step problem.

1. You find H, the height of the house.
2. Then you calculate the area of the façade. Use the height of the house and the width of the base of the house.

The solution is detailed below.



An A-Frame House

To find H, you must use the Tangent function, since you are dealing with the opposite and adjacent side to the angle.

The angle we will use in the solution is **half of the total included angle (65/2 = 32.5°)**. The length of the opposite side to the angle of 32.5° is 4.5m.

$$\tan(32.5) = \frac{opposite}{adjacent}$$

$$\tan(32.5) = \frac{4.5m}{H}$$

$$H = \frac{4.5m}{\tan(32.5)}$$

$$H = 7.06m$$

You now know that H is 7.06m. By using this dimension and **half the width of the base (4.5m),** you can calculate the area to be sided.

$$Area = \frac{base \times height}{2}$$

$$Area = \frac{4.5m \times 7.06m}{2}$$

$$Area = 15.89m^2$$

This $15.89m^2$ area is only half of the front façade. To get the full area, multiply this number by 2.

$$TotalArea = 15.89m^2 \times 2$$

$$TotalArea = 31.79m^2$$

So if you order $31.79m^2$ of siding for one façade, you should have more than enough, since some will be left over because of windows and doors.

Remember!

$$\text{Tan A} = \frac{\text{opposite side of } \angle \text{A}}{\text{Adjacent side of } \angle \text{A}}$$

Finding the angle of inclination (using the tangent)

You're a ground crew operator preparing for the launch of a rocket carrying satellite equipment. You are working 1,200 feet from the launch pad. Your task is to measure the time it takes from lift-off till the rocket reaches an approximate height of 2000 feet. By doing this, you can get a good estimate of the velocity of the rocket by dividing the distance that it traveled by the amount of time that it took.

From where you are standing, what is the angle of inclination of your line of sight from the horizontal when the rocket reaches 2000 feet?

Since you are relating the opposite side of the angle to the adjacent side, you must use the tangent function to solve this problem.

$$\tan(A) = \frac{opposite}{adjacent}$$

$$\tan(A) = \frac{2000\,ft}{1200\,ft}$$

$$\tan(A) = 1.666$$

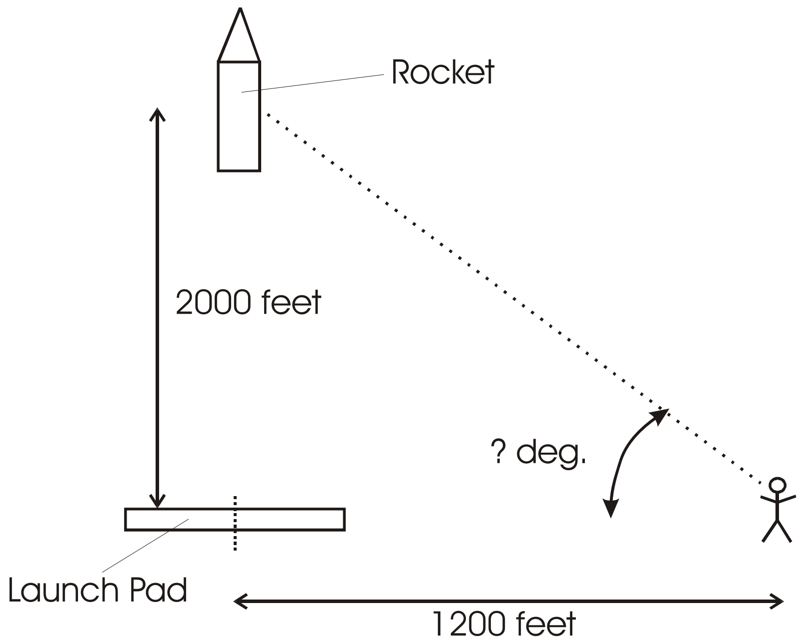
$$A = \tan^{-1}(1.666)$$

$$A = 59.04°$$

The angle of inclination of your line of sight is therefore 59.04°.

Remember!

$$\text{Tan A} = \frac{\text{opposite side of } \angle \text{A}}{\text{Adjacent side of } \angle \text{A}}$$



Using the Cosine Function

When Do You Use the Cosine Function?

You use the cosine when you are relating an acute angle in a right triangle, the side adjacent to that angle and the hypotenuse.

Much like the sine function, the cosine function is a ratio of the length of the sides of a right triangle.

The cosine of an angle is equal to the ratio of the length of the side adjacent to the length of the hypotenuse.

The cosine of angle A is the measure of the adjacent side to angle A divided by the measure of the hypotenuse.

The cosine of angle B is the measure of the adjacent side to angle B divided by the measure of the hypotenuse.

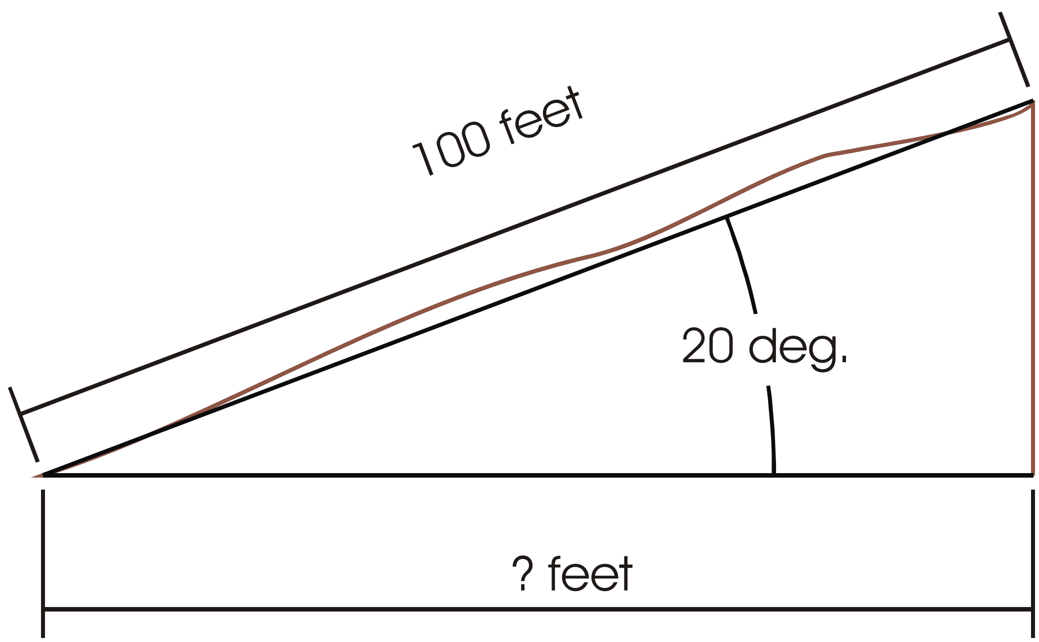
$$\text{Cos } A = \frac{\text{adjacent side of } \angle A}{\text{hypotenuse}} \qquad b/c$$

Cos B = adjacent side of ∠ B / hypotenuse a/c

Finding the adjacent side to the angle (using the Cosine Function)

You are managing a construction site and have been asked to design a ramp of gravel so that dump trucks are able to travel in and out of the elevated site.

Your first design involves an angle of 20° and a length along the slope of 100 feet. What will the horizontally projected distance be?



Cos A = adjacent side of ∠ A / hypotenuse

$\cos 20° = x / 100$ feet

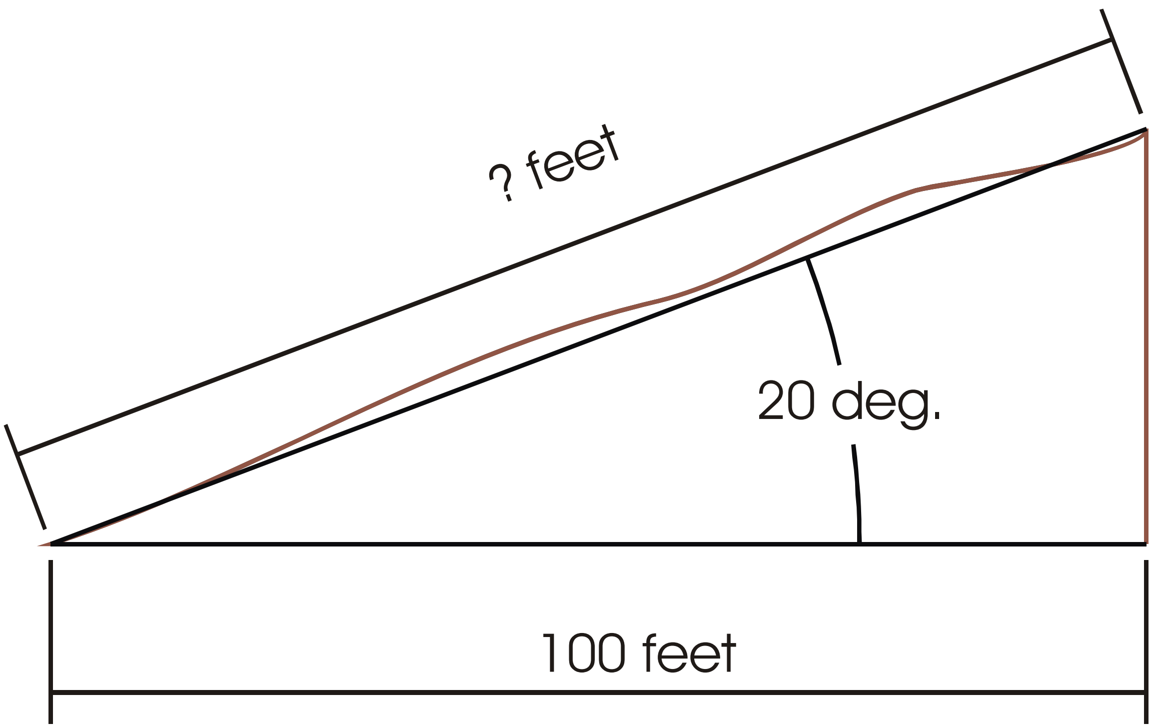
$100 (\cos 20°) = x$

93.97 feet $= x$

The horizontally projected distance is 93.97 feet.

Finding the hypotenuse (using the Cosine Function)

You then decide that an angle of 20° is reasonable, but you would like the horizontally projected distance to be 100 feet. What will the length along the slope be?



Cos A = $\frac{\text{adjacent side of } \angle A}{\text{hypotenuse}}$

cos 20° = 100 feet / y feet

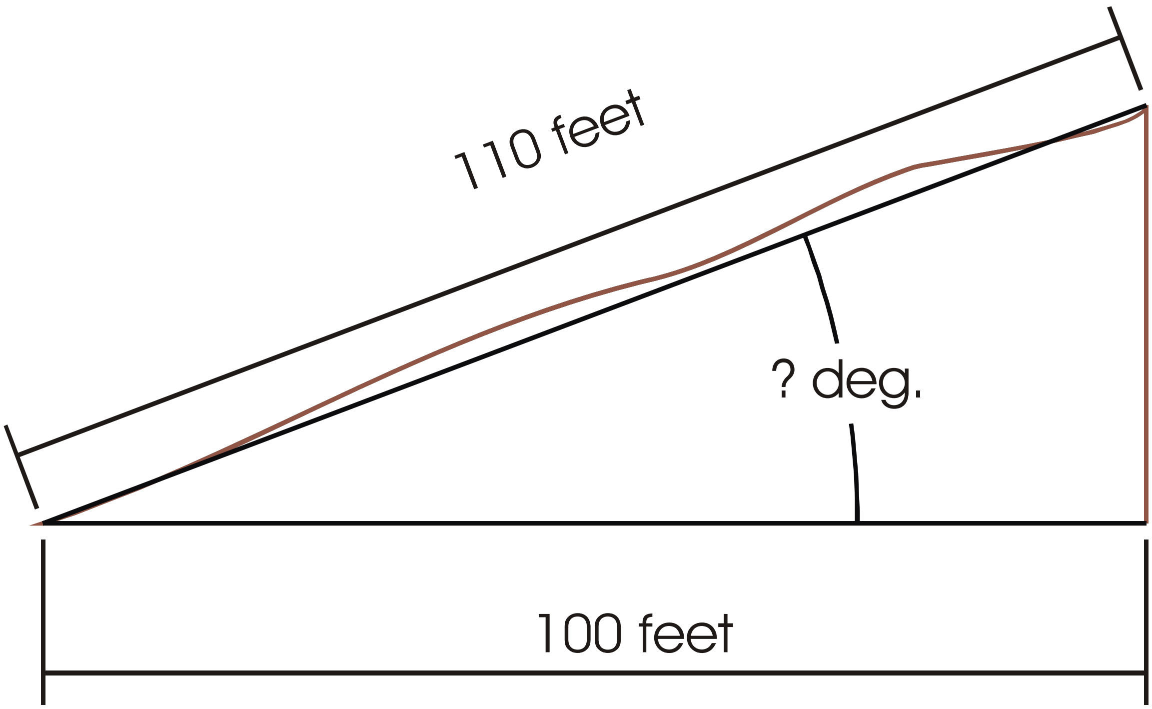
y = 100 feet / (cos 20°)

y = 106.42 feet

The length along the slope is 106.42 feet.

Finding the angle (using the Cosine Function)

As a final option you decide that the angle is not as important and so you specify that the length along the slope should be 110 feet and the horizontally projected distance should be 100 feet. What is the angle of inclination?



$$\cos A = \frac{\text{adjacent side of } \angle A}{\text{hypotenuse}}$$

cos A° = 100 feet / 110 feet

cos A° = 0.909

$A = \cos^{-1}(0.909)$

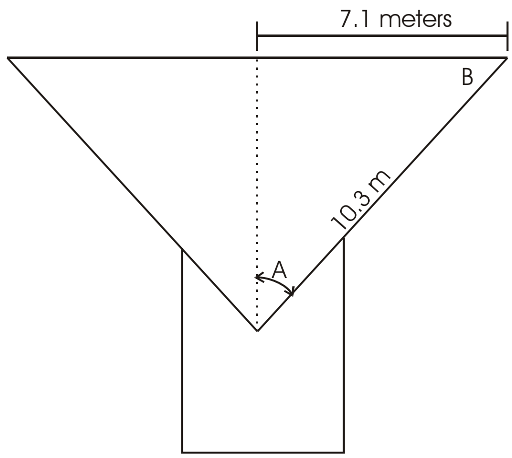
A = 24.62°

The angle of inclination is 24.62°.

Final Checkpoint

As an Environmental Engineer, you have been asked to design a conical reservoir for a wastewater treatment plant. You want to optimize the amount of material used while maintaining a sufficient volume-carrying capacity. You therefore decide to consider several dimensions.

You first design a storage basin with a radius of 7.1 meters and a sloped side of 10.3 meters. You need to find the included angles of the cone (triangle).



1. Find the value of A in this triangle.

a. 7.1^o

b. 43.58^o

c. 10.3^o

d. 7.1 meters

Answer is: b

Sin A = 7,1/10,3

A = $\sin^{-1}$ (7,1/10,3)

A ≅ $43,58^o$

2. Find the value of B in this triangle.

a. 34,85^o

b. 34,58 ^{o}

c. 46,52^o

d. 43,58^o

Answer is : c

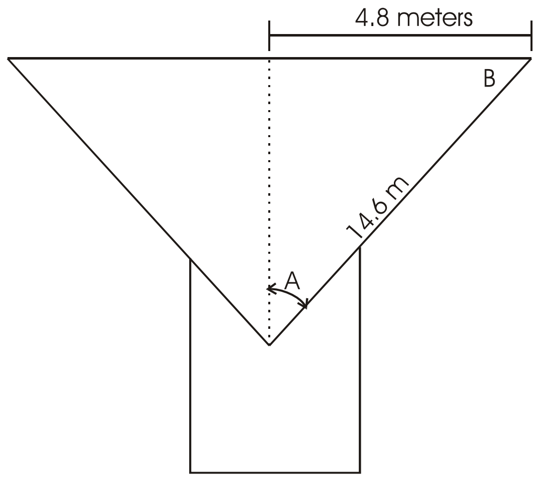
How?

$43,58^o + 90^o + B = 180^o$

$B = 180^o – 90^o – 43,58^o$

$B = 46,52^o$

You then try other dimensions. You choose a radius of 4.8 meters and a slope of 14.6 meters.



3. Find the value of A in this triangle.

a. 14.16^o

b. 16.14^o

c. 18.19^o

d. $19{,}19^o$

Answer is: d

How?

Sin A = 4,8 /14,6

A = $\sin^{-1}$ (4,8/14,6)

A ≅ $19{,}19^o$

From your years of Engineering schooling, you say to yourself "What would the included angle be if the ratio of the opposite side to the hypotenuse (sloped length) was 0.7880?"

4. If sin A = 0,7880, find angle A.

a. 0,0138

b. 0,2130

c. 52^o

d. 38^o

Answer is: c (52^o)

5. In which of these situations would you use the tangent function?

a. You are given the length of the adjacent side and opposite side and are looking to find the angle.

b. You are given the angle and the length of the opposite side and are looking for the length of the hypotenuse.

c. You are given the angle and the length of the hypotenuse and are looking for the length of the adjacent side.

d. You are given the length of the adjacent side and the hypotenuse and are looking for the angle.

Answer is: a

6.. In which of these situations would you use the cosine function?

a. You are given the length of the adjacent side and opposite side and are looking to find the angle.

b. You are given the angle and the length of the opposite side and are looking for the length of the hypotenuse.

c. You are given the angle and the length of the hypotenuse and are looking for the length of the adjacent side.

d. You are given the length of the opposite side and the hypotenuse and are looking for the angle.

Answer is: c

Glossary

Acute angle: An acute angle has an arc under 90^o.

Adjacent side: The adjacent side to an angle in a right triangle is the side, along with the hypotenuse, which forms one of the rays bounding the angle under consideration.

Angle of friction: The angle of friction of a material is the angle at which the material can remain stable under its own weight.

For instance, if a pile of sand is made, if the inclination of a side of the pile is greater than 40°, the sand will simply slide to the bottom of the pile.

Cable-stayed bridge: a type of suspension bridge in which the supporting cables are connected directly to the bridge deck without the use of suspenders

Convention: A convention is an established practice.

Cosine: the ratio between the leg adjacent to an angle when it is considered part of a right triangle and the hypotenuse.

Function: a mathematical relation such that each element of one set is associated with at least one element of another set.

Hydraulic engineer: an engineer who deals with the use and control of water in motion and the design of any structure/facility to accommodate the fluid.

Hypotenuse: The hypotenuse is the longest side of a right triangle.

Included angle: An acute angle which is bounded by two rays extending from the origin of the angle.

Obtuse angle: An obtuse angle has an arc over 90^o.

Opposite side: The opposite side to an angle in a right triangle is the side directly opposite to the angle under consideration.

Pythagorean Theorem: The square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides Bottom of Form.

Ratio of measures: A ratio of measures is the quotient, the proportion between two measures.

Right angle: A right angle is an angle with an arc of 90^o.

Right triangle: A right triangle has one angle of 90^o.

Sine: the ratio between the leg opposite to an angle when it is considered part of a right triangle and the hypotenuse.

Tangent: the ratio between the leg opposite to an angle when it is considered part of a right triangle and the leg adjacent.

Trigonometric ratios: formulas used to solve right triangles that find a missing measurement.

Trigonometry: the study of the properties, measurement and applications of triangles.

Keywords

Trigonometry,mathematics,triangles,angles,measurement,measures,calculations, sine,tangent, cosine, Hydraulic engineer, Pythagorean Theorem