

CORRECTION KEY Math 536.05

Part A

Questions 1 to 10 *4 marks or 0 marks*

- | | | | |
|----------|---|-----------|---|
| 1 | C | 6 | D |
| 2 | A | 7 | C |
| 3 | B | 8 | D |
| 4 | C | 9 | B |
| 5 | A | 10 | B |

Part B

Questions 11 to 15 *4 marks each*

- 11** The coordinates of the foci are **(1, -1)** and **(1, -7)**. *4, 2 or 0 marks* /4
- Allot 2 marks for each ordered pair.*
- 12** $f^{-1}(x) = \frac{8 + 2x}{x - 1}$ or $f^{-1}(x) = \frac{10}{x - 1} + 2$ *4 marks or 0 marks* /4
- 13** The equations of the asymptotes are **x = -4** and **y = 1**. *4, 2 or 0 marks* /4
- Allot 2 marks for each equation.*

14

Example of an appropriate solution

/4

$$\begin{aligned}
2 \cos^2 \theta - 3 \sin \theta &= 3 \\
2(1 - \sin^2 \theta) - 3 \sin \theta &= 3 && 1 \text{ mark} \\
2 - 2 \sin^2 \theta - 3 \sin \theta &= 3 \\
2 \sin^2 \theta + 3 \sin \theta + 1 &= 0 \\
(2 \sin \theta + 1)(\sin \theta + 1) &= 0 \\
\sin \theta = \frac{-1}{2} \quad \text{or} \quad \sin \theta = -1 &&& 1 \text{ mark}
\end{aligned}$$

For $\sin \theta = \frac{-1}{2}$	For $\sin \theta = -1$
Reference angles: $\theta = \frac{\pi}{6}$	$\theta = \frac{3\pi}{2}$ 1 mark

The sine function is negative in Quadrants III and IV.

Answer: In radian measure, the exact answers are $\theta \in \left\{ \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6} \right\}$ 1 mark

Note: *Correct answers in degree measure, or rounded numerical answers receive a maximum of 2 marks.*

15

Example of an appropriate solution

/4

$$\begin{aligned}
\vec{u}(2\vec{u} \cdot 3\vec{v}) &= \vec{u}(2(-1, 1) \cdot 3(1, 2)) \\
&= \vec{u}((-2, 2) \cdot (3, 6)) && 1 \text{ mark} \\
&= \vec{u}((-2 \cdot 3) + (2 \cdot 6)) \\
&= \vec{u}((-6) + (12)) \\
&= \vec{u}(6) && 1 \text{ mark} \\
&= 6\vec{u} \\
&= 6(-1, 1) \\
&= (-6, 6) && 2 \text{ marks}
\end{aligned}$$

Answer: The components of $\vec{u}(2\vec{u} \cdot 3\vec{v})$ are **(-6, 6)**.

Part C

Questions 16 to 25 4 marks each

No marks are to be given if work is not shown. Examples of correct solutions are given.

However, other acceptable solutions are possible.

16

Example of an appropriate solution

/4

x : number of almond chocolate bars
 y : number of caramel chocolate bars

Constraints

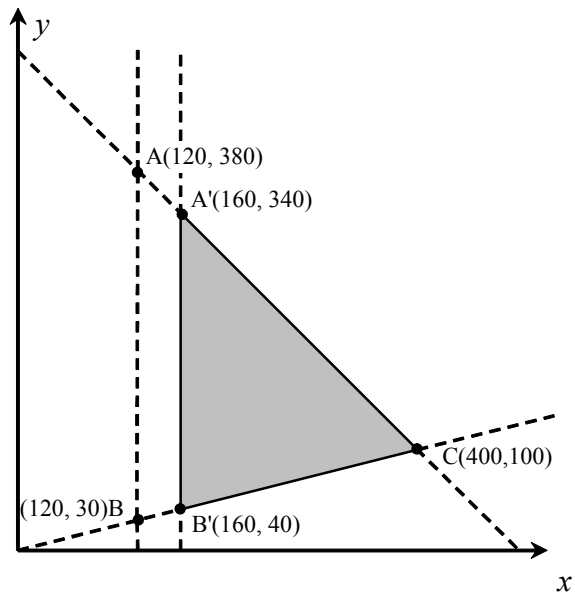
$$x + y \leq 500$$

$$x \leq 4y$$

$$x \geq 120$$

Profit

$$P = 0.8x + y$$



Point	Profit
A(120, 380)	\$476
B(120, 30)	\$126
C(400, 100)	\$420

Point	Profit
A'(160, 340)	\$468
B'(160, 40)	\$168
C'(400, 100)	\$420

Difference in profit

$$\$476 - \$468 = \$8$$

Answer: The difference in the maximum profit is \$8.

Note: Students who use an appropriate method in order to determine the constraints, graph the polygon and find the original corner points have shown they have a partial understanding of the problem.

Example of an appropriate solution

Greatest integer function

$$\begin{aligned}
 x = 0 \Rightarrow y &= 32.5[0.05(0) + 3] + 52.5 \\
 &= 32.5[3] + 52.5 \\
 &= 150 \text{ cm}
 \end{aligned}$$

$$\text{Step length} = \frac{1}{0.05} = 20 \Rightarrow \text{Last open point is } (20, 150)$$

Rational function

$$\begin{aligned}
 y &= \frac{a}{x + 10} - 10 \\
 150 &= \frac{a}{20 + 10} - 10 \\
 160 &= \frac{a}{30} \\
 a &= 4800
 \end{aligned}$$

Equation

$$y = \frac{4800}{x + 10} - 10$$

$$\begin{aligned}
 x = 180 \Rightarrow y &= \frac{4800}{180 + 10} - 10 \\
 &\approx 15.26 \text{ cm}
 \end{aligned}$$

Answer: To the nearest tenth of a centimetre, the distance is **15.3** cm.

Note: Students who use an appropriate method in order to correctly determine the point (20, 150) have shown they have a partial understanding of the problem.

Find the equation of the absolute value function

Vertex (1, 10) point (0, 2)

$$y = a|x - 1| + 10$$

$$2 = a|-1| + 10$$

$$-8 = a$$

$$y = -8|x - 1| + 10$$

Find the x value when $y = 1$

$$y = -8|x - 1| + 10$$

$$1 = -8|x - 1| + 10$$

$$-9 = -8|x - 1|$$

$$\frac{9}{8} = |x - 1|$$

$$\begin{aligned} \therefore x - 1 &= \frac{-9}{8} && \text{or} && x - 1 &= \frac{9}{8} \\ x &= \frac{-1}{8} && && x &= \frac{17}{8} \\ x &= \frac{17}{8} && \text{or} && 2.125 \end{aligned}$$

Find the equation of the square root function

Starting point (2.125, 1) point (3.125, 3)

$$y = a\sqrt{x - 2.125} + 1$$

$$3 = a\sqrt{3.125 - 2.125} + 1$$

$$2 = a\sqrt{1}$$

$$2 = a$$

So $y = 2\sqrt{x - 2.125} + 1$

Find the time when the height is 5 m

$$y = 2\sqrt{x - 2.125} + 1$$

$$5 = 2\sqrt{x - 2.125} + 1$$

$$4 = 2\sqrt{x - 2.125}$$

$$2 = \sqrt{x - 2.125}$$

$$4 = x - 2.125$$

$$6.125 = x$$

Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.

Note: Students who use an appropriate method in order to determine the starting point of the square root function have shown they have a partial understanding of the problem.

Do not penalize students who rounded their final answer.

Example of an appropriate solution

Let t : number of days
 $f(t)$: amount of the compound remaining (g)

$$f(t) = 150c^t$$

$$123 = 150c^{10}$$

$$0.82 = c^{10}$$

$$c = \sqrt[10]{0.82}$$

$$c \approx 0.98$$

Time for 75 g to remain:

$$f(t) = 150(0.98)^t$$

$$75 = 150(0.98)^t$$

$$0.5 = (0.98)^t$$

$$t = \log_{0.98} (0.5)$$

$$t = \frac{\log (0.5)}{\log (0.98)}$$

$$t \approx 34.3$$

Answer: To the nearest day, half of the compound will remain after **34** days.

Note: Students who do not round $\sqrt[10]{0.82}$ will obtain a rounded answer of 35 days. Accept answer of 34 or 35 days if appropriate work is shown.

Students who use an appropriate method in order to correctly determine the value $c \approx 0.98$ have shown they have a partial understanding of the problem.

Students who use the half-life formula $N = N_0 \left(0.5\right)^{\frac{t}{H}}$ where H is the half-life, and who obtain $\frac{10}{H} = \log_{0.5}(0.82)$ have shown they have a partial understanding of the problem.

Do not penalize students who did not round their final answer or rounded incorrectly.

$$a + c = 167$$

$$2c = 132$$

$$\therefore c = 66 \text{ and } a = 101$$

$$b^2 = a^2 - c^2$$

$$b = \sqrt{101^2 - 66^2}$$

$$b \approx 76.45$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{10\,201} + \frac{y^2}{5845} = 1$$

$$\frac{66^2}{10\,201} + \frac{y^2}{5845} = 1$$

$$y \approx 57.87$$

Answer: To the nearest length, the distance from Mars to the Sun is **57.9** AU.

Note: Students who use an appropriate method in order to determine parameters a , b and c have shown they have a partial understanding of the problem.

Do not penalize students who did not round their answers or rounded incorrectly.

Circle

Centre: (13, 10) radius: 4 cm

End points of the diameter
(9, 10) and (17, 10)

Hyperbola

The vertices are (9, 10) and (17, 10) and therefore $a = 4$.

Half of the total length is 7 cm. So the foci are (6, 10) and (20, 10) and therefore $c = 7$.

Equation of hyperbola

$$c^2 = a^2 + b^2$$

$$7^2 = 4^2 + b^2$$

$$33 = b^2$$

$$\therefore \frac{(x - 13)^2}{16} - \frac{(y - 10)^2}{33} = 1$$

To find the height let $x = 6$ and find the y coordinate

$$\frac{(6 - 13)^2}{16} - \frac{(y - 10)^2}{33} = 1$$

$$33(-7)^2 - 16(y - 10)^2 = 16(33)$$

$$1617 - 528 = 16(y - 10)^2$$

$$1089 = 16(y - 10)^2$$

$$68.0625 = (y - 10)^2$$

$$\pm 8.25 = (y - 10)$$

$$\text{So } \begin{matrix} y = 10 + 8.25 \\ = 18.25 \end{matrix} \quad \text{and} \quad \begin{matrix} y = 10 - 8.25 \\ = 1.75 \end{matrix}$$

Answer: The height of the frame is **16.5** cm.

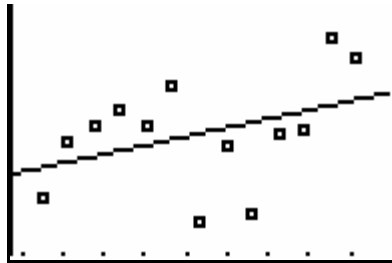
Note: Students who use an appropriate method in order to determine a correct equation of the hyperbola have shown they have a partial understanding of the problem.

Example of an appropriate solution

Let x : year
 y : number of homicides

Using a graphing calculator

Calculator method



LinReg

$$y = ax + b$$

$$a = 5.631868132$$

$$b = -10527.04396$$

$$r^2 = 0.1566628593$$

$$r = 0.3958065933$$

$$y_1(2002)$$

$$747.9560403$$

Note: Students may obtain 624.0549451 as the value of parameter b if they use the number of years elapsed since 1980 as the independent variable.

Without a graphing calculator

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{732 - 592}{1992 - 1980}$$

$$= 11.7$$

$$y = 11.7x + b$$

$$732 = 11.7(1992) + b$$

$$b = -22\,574$$

$$y = 11.7x - 22\,574$$

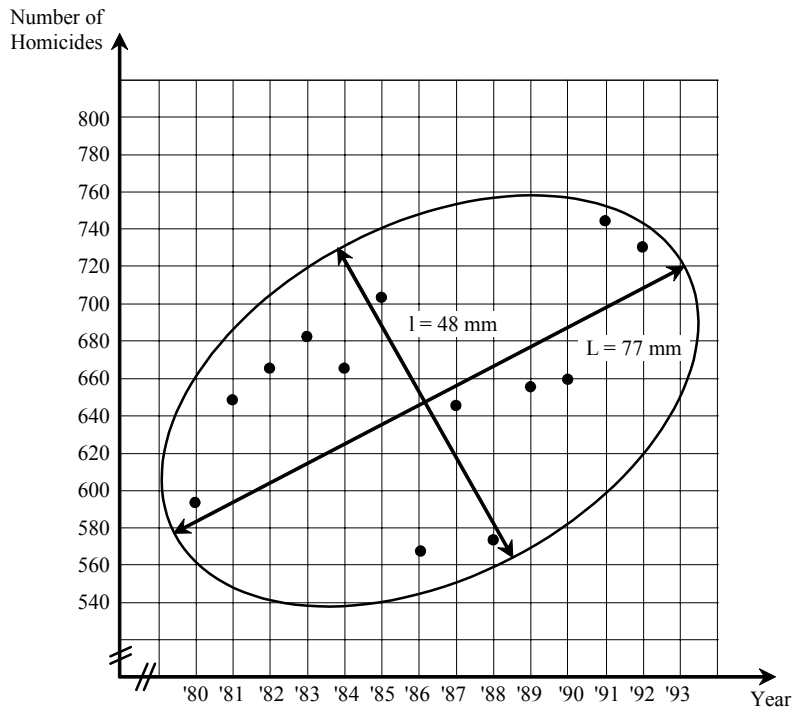
$$y = 11.7(2002) - 22\,574$$

$$y = 849$$

$$r = \left(1 - \frac{48}{77}\right)$$

$$\approx 0.38$$

(Student values will vary)



Therefore, it would be expected that approximately 748 (849 or equivalent using a graph) homicides would occur in 2002.

Answer: Explanation: 582 homicides are not consistent with the model, which would suggest significantly more homicides than actually occurred. Since the correlation coefficient is low (0.40), the model itself is not a reliable predictor.

Note: Students who use an appropriate method in order to determine a correct equation for the line of regression have shown they have a partial understanding of the problem.

Example of an appropriate solution

$\angle ABC = 90^\circ$ An angle inscribed in a semi-circle is a right angle

Length of \overline{AC}

$$5(m \overline{AC}) = 13^2 \quad \text{proportional mean}$$

$$5(m \overline{AC}) = 169$$

$$m \overline{AC} = 33.8 \text{ cm}$$

Length of \overline{ED}

$$m \overline{ED} + 14 + 5 = 33.8 \quad \text{length of the diameter}$$

$$m \overline{ED} = 14.8 \text{ cm}$$

Length of \overline{EB}

$$m \overline{EB}^2 = 12^2 + 14.8^2$$

$$m \overline{EB} = 19.05 \text{ cm}$$

Length of \overline{EF}

$$m \overline{EF}(19.05) = 14(33.8 - 14) \quad \text{constant product theorem}$$

$$m \overline{EF}(19.05) = 14(19.8)$$

$$m \overline{EF} = 14.55 \quad (14.548 \text{ if you use } 19.053 \text{ } 608 \text{ } 58)$$

Answer: To the nearest tenth, the measure of \overline{EF} is **14.6** cm.

Note: Students who use an appropriate method in order to determine the length of the diameter have shown they have a partial understanding of the problem.

Do not penalize students who did not round their answers or rounded incorrectly.

$$\begin{aligned} (m \overline{AB})^2 &= (m \overline{AF}) \times (m \overline{AE}) \\ 16^2 &= 8 m \times (m \overline{AE}) \\ m \overline{AE} &= 32 \text{ cm} \end{aligned}$$

Segment \overline{DE} is a tangent and segment \overline{OE} is a radius, so $m \angle DEA = 90^\circ$.

In $\triangle AED$

$$\begin{aligned} (m \overline{AD})^2 &= (m \overline{AE})^2 + (m \overline{DE})^2 \\ (m \overline{AD})^2 &= 32^2 + 24^2 \\ (m \overline{AD})^2 &= 1600 \\ m \overline{AD} &= 40 \text{ cm} \end{aligned}$$

$$\begin{aligned} (m \overline{AD})(m \overline{EC}) &= (m \overline{DE})(m \overline{AE}) \\ 40(m \overline{EC}) &= (24)(32) \\ m \overline{EC} &= \frac{768}{40} \\ m \overline{EC} &= 19.2 \text{ cm} \end{aligned}$$

Answer: The measure of \overline{EC} is **19.2** cm.

Note: Students who use an appropriate method in order to correctly determine $m \overline{AE}$ have shown they have a partial understanding of the problem.

The minimum height of the pendulum is 150 cm.

The maximum height of the pendulum is $150 + (31 - 31 \sin 45^\circ) = 159.08$ cm.

$$y = a \cos b(x - h) + k$$

$$a = \frac{159.08 - 150}{2}$$

$$= 4.54$$

$$b = \frac{2\pi}{0.875}$$

$$= \frac{16\pi}{7}$$

$$k = \frac{159.08 + 150}{2}$$

$$= 154.54$$

$h = 0$ if a is negative

$$f(t) = -4.54 \cos\left(\frac{16\pi t}{7}\right) + 154.54$$

1 hour = 3600 seconds

$$f(3600) = 156 \text{ cm}$$

Answer: To the nearest cm, the pendulum will be **156** cm above the ground after 1 hour.

Note: Students who use an appropriate method in order to determine parameters a , b and k have shown they have a partial understanding of the problem.

Students' answers may vary considerably, depending on the rounding they applied in the course of