Magnitude and Direction of a Vector

A vector is a quantity that has both ***magnitude*** and ***direction***. (Magnitude = size)

Looking for some real life Vector Quantities??...

- You travel 30 km in a Northerly direction (magnitude is 30 km, direction is North - this is a *displacement vector*)

- The train is going 80 km/h towards Toronto (magnitude is 80 km/h, direction is towards Toronto - it is a *velocity vector)*

- My friend Aaron (A Civil Engineer no less!) says the force on a bridge is 50 N acting downwards (the magnitude is 50 Newtons and the direction is down - it is a *force vector*)

More vector examples….

Acceleration, momentum, angular momentum, magnetic and electric fields

Each of the examples above involves **magnitude and direction**.

Remember: A vector is not the same as a **scalar**. (see Part 1 on MiLearn.com) . Scalars have **magnitude only**. For example, a **speed** of 35 km/h is a scalar quantity, since no direction is given. Other examples of *scalar quantities* are:

Volume, density, temperature, mass, speed, time, length, distance, work and energy.

Each of these quantities has magnitude only, and do not involve direction.

Vector Notation

Look for bold **capital letters** to name vectors. For example, a force vector could be written as **F**.

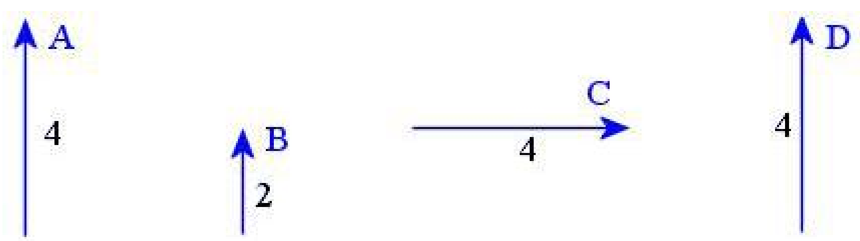
Alternative vector notations

- Some textbooks write vectors using an arrow above the vector name, like this: $\vec{\mathbf{F}}$
- You will also see vectors written using matrix-like notation. For example, the vector acting from (0, 0) in the direction of the point (2, 3) can be written

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

A vector is drawn using an **arrow**. The **length** of the arrow indicates the **magnitude** of the vector. The **direction** of the vector is represented by (not surprisingly :-) the direction of the arrow.

Examples -Vectors



The displacement vector **A** has direction 'up' and a magnitude of 4 cm.

Vector **B** has the same direction as **A**, and has half the magnitude (2 cm).

Vector **C** has the same magnitude as **A** (4 units), but it has different **direction**.

Vector **D** is equivalent to vector **A**. It has the same magnitude and the same direction. It doesn't matter that **A** is in a different position to **D** - they are still considered to be **equivalent vectors** because they have the same magnitude and same direction. We can write:

A = **D**

FYI: You cannot write **A** = **C** because even though **A** and **C** have the same magnitude (4 cm), they have different direction. They are not equivalent.

Free & Localized Vectors

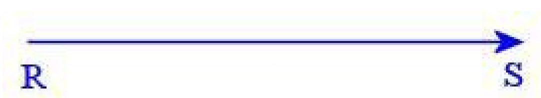
So far we have seen examples of "free" vectors. We draw them without any fixed position.

Another way of representing vectors is to use **directed line segments.** This means the vector is named using an **initial point** and a **terminal point**. Such a vector is called a *"localized vector"*.

Examples of Localized Vectors

A vector **OP** has initial point **O** and terminal point **P**. When using directed line segments, we still use an arrow for the drawing, with **P** at the arrow end. The length of the line OP is an indication of the magnitude of the vector.

We could have another vector **RS** as follows. It has initial point **R** and terminal point **S**.



Because the 2 vectors have the same magnitude and the same direction (they are both horizontal and pointing to the right), then we say they are equal. We would write:

OP = **RS**

Note that we can move vectors around in space and as long as they have the same vector magnitude and the same direction, then they are considered **equal vectors**.

Magnitude of a Vector

We indicate the **magnitude** of a vector using **vertical lines** on either side of the vector name.

The magnitude of vector **PQ** is written |**PQ**|.

Think of having vector **A** with a magnitude of 4 units. We would write the magnitude of vector **A** as:

| **A** | = 4

Scalar Quantities

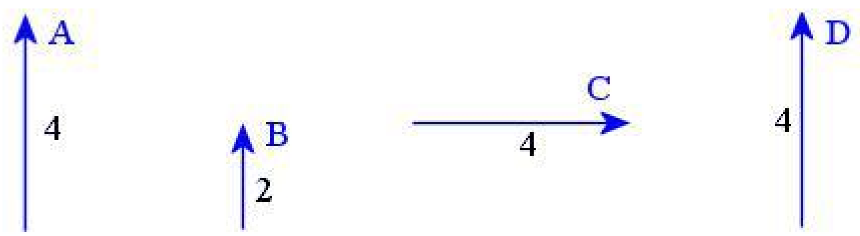
A **scalar quantity** has **magnitude**, but not direction.

For example, *a pen may have length "10 cm".* The length 10 cm is a **scalar quantity** - it has magnitude, but no direction is involved.

Scalar Multiplication

We can increase or decrease the magnitude of a vector by multiplying the vector by a scalar.

Scalar Multiplication – Ex. 1



Note that vector **B** (2 units) is half the size of vector **A** (which is 4 units). You can say:

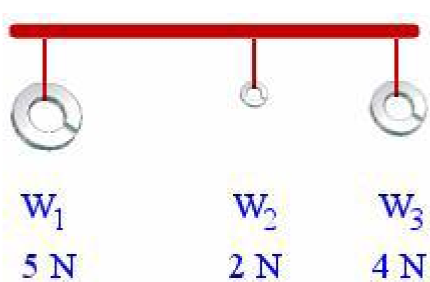
B = 0.5 **A**

This is a scalar multiple. We have multiplied the vector **A** by the scalar 0.5.

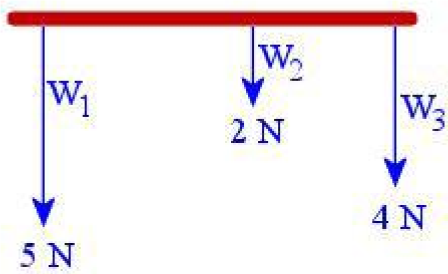
Scalar Multiplication – Ex. 2

We have 3 weights tied to a beam.

The first weight is $\mathbf{W_1}$ = 5 N, the second is $\mathbf{W_2}$ = 2 N and the third is $\mathbf{W_3}$ = 4 N.



We can represent these weights using a vector diagram (where the length of the vector represents the magnitude) as follows:



They are vectors because they all have a direction (down) and a magnitude.

Each of the following scalar multiples is true for this situation:

Since $5 = 2.5 \times 2$, we can write:

$\mathbf{W_1} = 2.5\ \mathbf{W_2}$

Since $2 = 0.5 \times 4$, we can write:

$\mathbf{W_2} = 0.5\ \mathbf{W_3}$

Since $4 = 0.8 \times 5$, we can write:

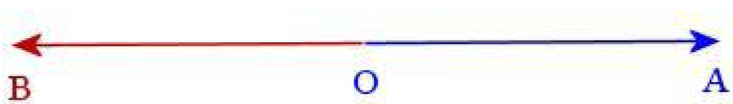
$\mathbf{W_3} = 0.8\ \mathbf{W_1}$

Each of these statements = a scalar multiplication.

Vectors in Opposite Directions

We have 2 university teams (Queens vs. University of Toronto – forgive the colours of the uniforms in the pic☺) playing a tug-of-war match. At the beginning of the game, they are very evenly matched and are pulling with equal force in opposite directions. We could name the vectors **OA** and **OB**.

We can represent the tug of war using a vector diagram:



The **magnitude** of each vector is the same, but they are acting in **opposite directions**. In this case, one must indicate the opposite directions by use of a **negative sign**.

So we write:

OA = −OB

Cool so far?????................

k…

Zero Vectors

A **zero vector** has magnitude of 0. It can have any direction.

A vector may have zero magnitude at an instance in time. Example = a boat bobbing up and down in the water will have a **positive velocity vector** when moving up, and a **negative velocity vector** when moving down. At the instant when it is at the top of its motion, the magnitude is **zero**.

In the tug-of-war example above, the teams are evenly matched at a certain instant and neither side is able to move. In this case, we would have:

OA + **OB** = **0**

The 2 force vectors **OA** and **OB**, operating in opposite directions, cancel each other out.

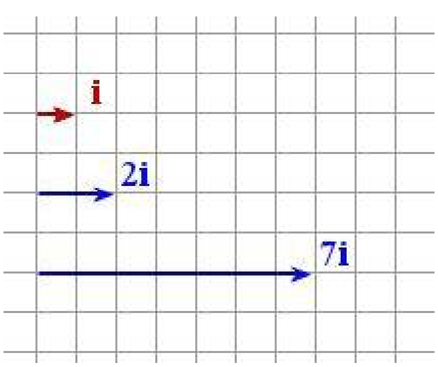
Unit Vectors

A **unit vector** has length **1 unit** and can take any direction.

A one-dimensional unit vector is usually written **i**.

Example - Unit Vector

In the following diagram, we see the **unit vector** (in red, labeled **i**) and two other vectors that have been obtained from **i** using scalar multiplication (2**i** and 7**i**).



Back to milearn.com